Functions & Their Representations

Understanding the Fundamental Concept in Calculus

Calculus and Analytical Geometry • Class Notes

What is a Function?

Definition: A function f from a set S to a set Y is a rule that assigns a unique value y in Y to each value x in S.

The set of all possible input values (x-values)

The set of all possible output values (y-values)

Notation: $y = f(x)$

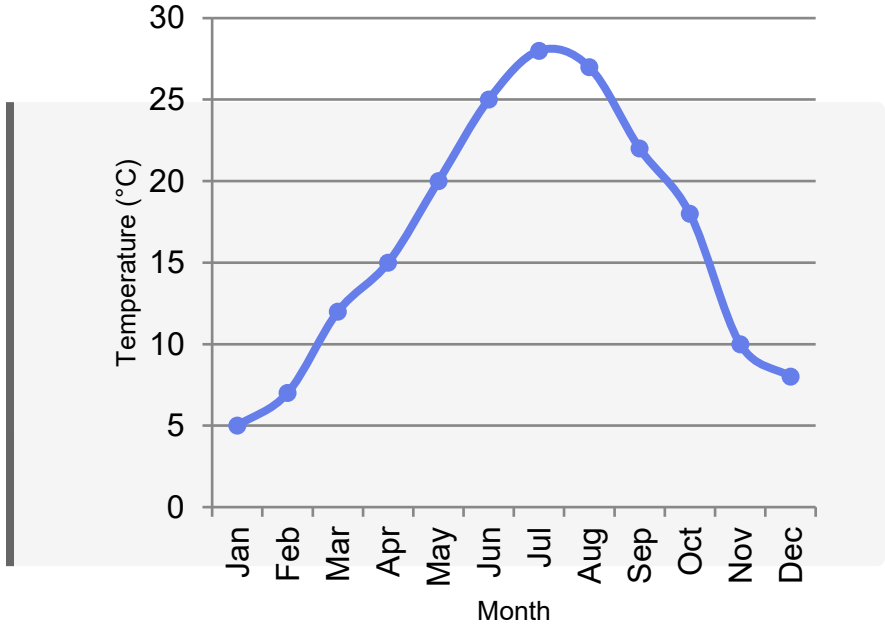
Real-World Example: Average Temperature

🌡️ How temperature depends on the month of the year

1 VERBAL (Words)

> "The average temperature of a city varies throughout the year, increasing from winter to summer and decreasing from summer to winter."

2 NUMERICAL (Table)



Real-World Example: Distance and Speed

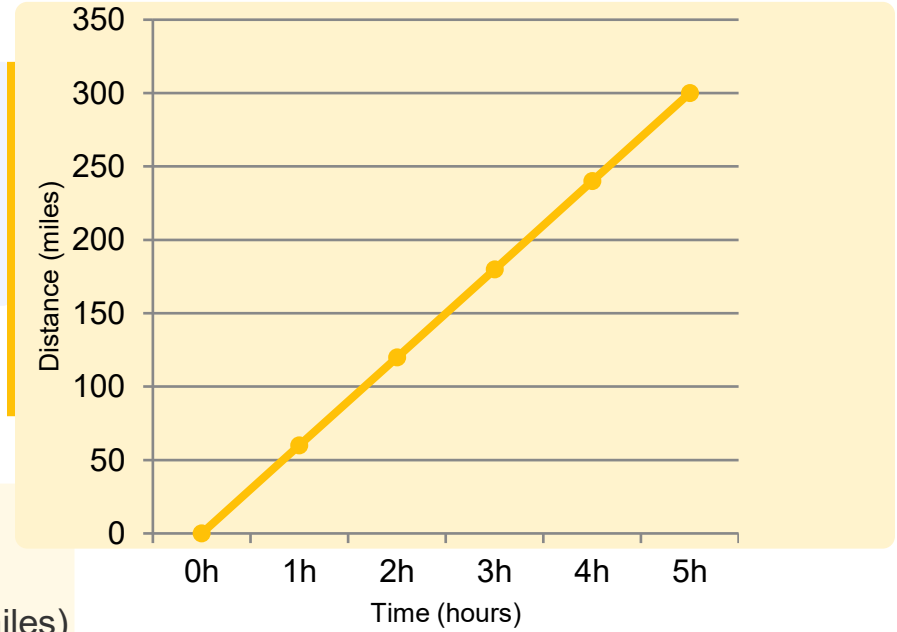
🚙 How distance traveled depends on time at constant speed

1 VERBAL

A car travels at 60 miles per hour. The distance traveled depends on how many hours it has been driving.

Key Insight: The distance is a function of time. We can calculate distance for any time value using the formula d = 60t.

3 ALGEBRAIC



$$d = 60t$$

- d = distance (miles)
- t = time (hours)
- 60 = speed (mph)

Real-World Example: Cell Phone Plans

🧮 Cost as a function of data usage

VERBAL

> A cell phone plan costs $30/month plus $5 per GB of data used.

ALGEBRAIC

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C = 30 + 5g
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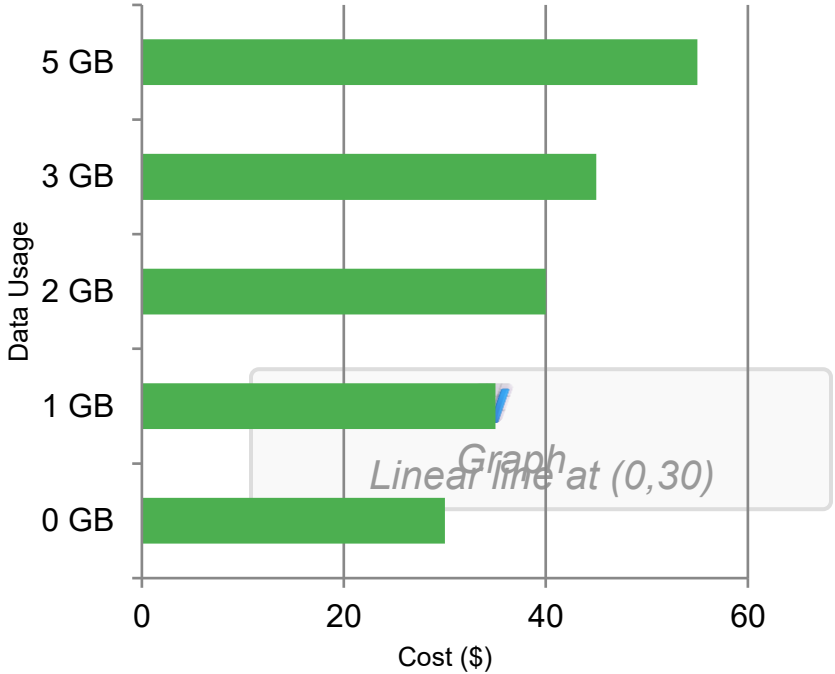
C = total cost ($)

g = data (GB)

NUMERICAL



The Four Ways to Represent a Function

1

Verbal

Describe the relationship in words

2

Numerical

Show values in a table

3

Algebraic

Express as a formula

4

Visual

Display on a graph

🗺️ Mapping Diagrams

A mapping diagram visually shows how elements from the domain map to the codomain using arrows.

Domain → Codomain

$x_1 \rightarrow f(x_1) = y_1$

$x_2 \rightarrow f(x_2) = y_2$

$x_3 \rightarrow f(x_3) = y_3$

✓ The Rule of Functions

Each input must have exactly ONE output arrow

Example: For f(x) = 2x:

- $1 \rightarrow 2$
- $2 \rightarrow 4$
- $3 \rightarrow 6$

✓ This IS a function

Each input has exactly one output

✨ Key Takeaways

- A function assigns exactly one output to each input
- Functions can be represented four ways: Verbally, Numerically, Algebraically, and Visually
- The domain is the set of inputs; the codomain is the set of outputs
- Notation: y = f(x) means "y is a function of x"
- Different representations are useful in different situations

> 💡 *Practice converting between all four representations to strengthen your understanding!*